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THERMAL RADIATION BETWEEN CLOSELY SPACED METAL SURFACES AT LOW TEMPERATURE DUE TO TRAVELING AND QUASI-STATIONARY COMPONENTS OF THE RADIATION FIELD

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Abstract—Radiant heat-transfer effects between closely spaced parallel metal surfaces at low temperature are analyzed. The theory considers thermally induced fluctuating electric fields at the surface of the metal, similar to those responsible for Nyquist noise, as the source of the thermal radiation field in the vacuum space between the metals. Both traveling wave and quasi-stationary wave components of the thermal radiation field are shown to exist in the vacuum region due to these sources. The electric and magnetic field vectors associated with these fields existing in the vacuum space are derived utilizing standard electromagnetic boundary theory, and the resulting unidirectional heat fluxes are calculated using the Poynting theorem. The resulting heat fluxes are shown to correspond to highly unclassical heat-transfer effects when the product of spacing distance l and the surface temperature T is less than 1 cm °K. At small spacing distances the heat flux first rises in a manner inversely proportional to the spacing distance and then where $lT \lesssim 10^{-2}$ cm °K, the unidirectional heat transfer rises inversely proportional to the fourth power of this product. The results of this theory are shown to be consistent with previous experimental measurements.

NOMENCLATURE

a, $= \frac{KTl}{\pi\hbar c} = 0{\cdot}404\frac{l}{\lambda_m}$ [nondimensional length];
c, velocity of light [cm/s];
C, $= \frac{\omega\mu\alpha_N(\omega)}{k^3} I_{0\omega}$, correlation function;
d, $= (2/\mu\sigma'\omega)^{1/2}$, skin depth [cm^{-1}];
e, electric charge [C];
$\boldsymbol{E}$, electric field vector;
h, Planck's constant [J-s];
$\hbar$, $= h/2\pi$;
$\boldsymbol{H}$, magnetic field vector;
$I_{0\omega}$, spectral blackbody intensity;
$\boldsymbol{k}$, $\frac{\boldsymbol{k}}{|\boldsymbol{k}|}\frac{2\pi}{\lambda}$ electromagnetic propagation vector [cm^{-1}];
k_1, k_2, k_3, Cartesian components of propagation vector;
K, Boltzmann's constant [J/°K];
l, spacing distance [cm];
m, electronic mass [kg];
N, electron density [e/m^3];
$P_q(k)$, spectral unidirectional radiative heat flux due to quasi-stationary waves [W/cm^2];
$\boldsymbol{P}_q$, total unidirectional heat flux due to quasi-stationary waves [W/cm^2];
$P_t(k)$, spectral unidirectional radiative heat flux per mode due to traveling waves [W/cm^2];
$\boldsymbol{P}_t$, total unidirectional heat flux due to traveling waves [W/cm^2];
$P(k)$, Poynting flux per mode [W/cm^2];
$\boldsymbol{P}$, total unidirectional heat flux [W/cm^2];
$\boldsymbol{P}_\infty$, total classical unidirectional heat flux [W/cm^2];
R_N, normal reflectance;
s, surface area [cm];
T, temperature [°K];
$\boldsymbol{U}$, total energy density for finite cavity [J/cm^3];
$\boldsymbol{U}_\infty$, total energy density for infinite cavity [J/cm^3];
v_F, Fermi velocity;
V, cavity volume [cm^3];
Z_0, $= \sqrt{\mu/\varepsilon}$, impedance of free space;
$\tilde{Z}$, surface impedance of a metal;
Z^*, $= \left(\frac{1}{1{\cdot}1547}\right)\left(\frac{2\pi}{\omega_p}\right)^{2/3}\left(\frac{v_F}{6\pi c}\right)^{1/3}$.

Greek letters

$\alpha_N(\omega)$, normal spectral absorptance of cavity wall;
δ, delta function;
ε, electrical capacitivity of free space [F/cm];
ε_H, hemispherical emittance;
$\Gamma(x)$, gamma function;
λ, wavelength of electromagnetic wave [cm];
λ_m, wavelength of maximum spectral intensity $= 2{\cdot}898 \times 10^{-3}/T$ [cm];
Λ, electron mean free path [cm];
μ, magnetic permeability of free space [H/cm];
ν, frequency of electromagnetic wave [s^{-1}];
σ, Stefan–Boltzmann constant [$W/cm^2\,{}^\circ K^4$];
σ', electrical conductivity [Ω^{-1}/cm];
ω, angular frequency of electromagnetic wave;
ω_p, $= (Ne/m\varepsilon)^{1/2}$, plasma frequency;
$\zeta(\alpha)$, Riemann zeta function.

INTRODUCTION

SEVERAL earlier papers have addressed the theory of nonclassical radiative heat-transfer effects between closely spaced metal surfaces at low temperature [1, 2, 3]. These analyses predicted radiation heat-transfer effects between closely spaced metal surfaces based on the assumption of an isotropic thermal radiative source field consisting of undamped plane waves interior to the metals and the use of standard electromagnetic boundary theory to predict the heat transfer through the calculation of a transmission coefficient between the metal surfaces through an intervening vacuum region [1, 3]. In these analyses both wave interference and tunneling effects are present [4]. Rytov has pointed out [5] that the concept of a thermal radiation field interior to a metal is invalid and that a proper analysis begins with the treatment of the thermally induced fluctuations of carrier or current density in the metal surfaces as the source of the exterior radiation field. This model leads to the concept of a corresponding thermally induced lateral electric or magnetic surface field as the source field for radiative heat transfer for a metal in which the skin effect is well developed. Polder and Van Hove [6] have published a generalized theoretical analysis of radiative heat transfer between metallic surfaces in the vein of Rytov [5] using a thermally induced surface current source term and a generalized model for the electrodynamic properties of the metal surface. These results depend in such a complex manner on the dielectric properties of the metal that they do not lend themselves to even qualitative conclusions on the variation of heat-transfer effects with material changes. However, Polder and Van Hove numerically calculate the spacing dependence of the radiative transfer between two chromium surfaces near room temperature and find semiquantitative agreement between their theory and the experimental measurements of Hargreaves [7]. They ascribe the differences between their theoretical predictions and Hargreaves measurements to their use of the dielectric properties of chromium based on the bulk measurements in contrast to the presumed differing dielectric properties of the thin chromium films used in Hargreaves experimental measurements. The present analysis addresses the problem of heat transfer between highly conductive metal surfaces at low temperature using the methods developed by Rytov where it is assumed that the extreme anomalous skin effect (EASE) theory will describe the electrodynamic properties of the metal and further that the skin effect is well developed, i.e. only a thin surface layer of the metal is responsible for the emission and absorption of radiation. Under these assumptions the dependence of the radiative heat transfer in spacing effect and electrodynamic properties of the metal are uncoupled and simple approximate expressions describe the heat-transfer effects and material dependence. The results of this theory are in good agreement with the experimental measurements of Cravahlo, Domoto and Tien [8], and Boehm and Tien [9] showing enhanced radiative heat transfer between closely spaced copper surfaces at low temperature.

The present theory is based on the recognition that the source of the thermal radiation in a highly conductive metal is due to fluctuations of the electrons in the metal due to temperature of the metal. The fluctuations in carrier density give rise to the presence, in particular, of corresponding lateral fluctuating electric fields having all spatial frequencies in the skin region which, by virtue of the continuity conditions imposed on the tangential component of the electric field demanded by Maxwell's equations, result in corresponding fields in the vacuum region surrounding the metal. An electromagnetic wave in the vacuum space may be characterized in part by its propagation vector $\boldsymbol{k}$, which will have spatial components k_1, k_2, k_3 satisfying the relationship that $|\boldsymbol{k}|^2 = k_1^2+k_2^2+k_3^2$, where the frequency of the electromagnetic wave is related to its propagation vector $\omega^2 = |\boldsymbol{k}|^2c^2$. For electromagnetic waves of a given frequency, $\omega^2 = |\boldsymbol{k}|^2c^2$ in the vacuum space for those spatial components k_1 and k_2 of the source field (taken to be those components in the plane of the surface) such that $k_1^2+k_2^2 \leqslant |\boldsymbol{k}|^2$, then the component of the propagation vector k_3 normal to the surface for the wave *in vacuo* will be real, thus corresponding to a traveling wave. However, if $k_1^2+k_2^2 > |\boldsymbol{k}|^2$, then the corresponding wave developed in the vacuum region will have an imaginary component for k_3 and hence will be confined to the region near the metallic surface. This wave is termed a quasi-stationary wave. The amplitudes of the traveling and quasi-stationary waves are determined by the strength of the source field. In the present analysis, the strength of the source field is determined through the requirement that at a large distance from the metallic surface the radiant emission observed in the vacuum region should be given by the classical value $\pi\varepsilon_{H\omega} I_{0\omega}$ when $\varepsilon_{H\omega}$ is determined by the electrodynamic properties of the metal. Once the amplitudes of the electromagnetic fields present in the vacuum region due to the source field have been determined, the heat transfer to a second surface is determined by use of Poynting's theorem. In this manner, it will be shown that both traveling and quasi-stationary waves play nonclassical roles in the heat transfer at small spacing distances and further that at small spacing distances the heat transfer due to quasi-stationary waves is the dominant heat-transfer mechanism.

METHOD OF APPROACH

Due to the finite surface temperature of the absorbing medium, i.e. the metal surface, a random electric field is present in the medium analogous to the electric fields responsible for Nyquist noise. This field is the result of macroscopic local polarizations occurring in the metal due to thermal agitation of the microscopic charges in the metal. Rytov [5] has shown that in a metallic medium in which the skin effect is well developed† this thermal agitation gives rise to a random lateral electric field K in which components with all frequencies are present, i.e.‡ §

$$\boldsymbol{K} = \iint_{-\infty}^{\infty} \boldsymbol{g}(k_1, k_2) \exp[i(k_1 x + k_2 y)]\, \mathrm{d}k_1\, \mathrm{d}k_2 \quad (1)$$

where the kernel $\boldsymbol{g}(k_1, k_2)$ of this integral is defined by the following correlation function:

$$\overline{g_\alpha(k_1', k_2') g_\beta(k_1'', k_2'')} = C\delta_{\alpha\beta}\delta(k_1' - k_1'')\delta(k_2' - k_2'') \quad (2a)$$

from which

$$\overline{K_\alpha(x', y') K_\beta(x'', y'')} = (2\pi)^2 C\delta_{\alpha\beta}\delta(x' - x'')\delta(y' - y'') \quad (2b)$$

where

$$C = \frac{\omega\mu\alpha_N(\omega)}{k^3} I_{0\omega}. \quad (3)$$

The bar on the right-hand sides of equations (2) represents a time or ensemble average. The correlation function C is determined by considering the thermal radiation emitted by an infinite metal surface due to the random lateral surface field and noting that at large distances from the surface the emitted radiant intensity should be given $\pi\varepsilon_{H\omega} I_{0\omega}$. The subscripts α and β in equations (2) refer to the two spatial dimensions in the plane of the metal surface, i.e. those denoted by the axes 1 and 2. The term $I_{0\omega}$ is the radiant intensity for blackbody radiation in a vacuum. The delta function correlation for the spatial coordinates occurs only because in deriving equations (2), Rytov averaged the microscopic electrodynamic equations over volumes sufficient to average out extreme fluctuations due to the graininess of the individual atomic charges.

This procedure provides a simple delta correlation function given by equation (1), which simply means that our results will be valid only up to spacing distances on the order of the correlation distance. Starting with the lateral field given by equation (1) for, say, the first medium and utilizing the electromagnetic boundary conditions at the two metal surfaces, the field in the vacuum space will be determined. Once the field in the vacuum space due to the lateral field of the first surface is determined, the thermal flux to the second surface due to this source will be determined by the application of Poynting's theorem at the second surface. The net heat transfer is simply the difference between the heat flux to the second surface due to the lateral field of the first surface, less the heat flux to the first surface produced by the lateral field of the second surface.

† Rytov has also provided a prescription for the correlation function where the skin effect is not fully developed, thus providing a basis for calculating the total radiative transfer between surfaces under conductions where the conductivity is not high and/or the frequencies of the electromagnetic radiation are high.

‡ The use of a correlation function that is only dependent on the coordinates in the plane of the surface is based on the following considerations. According to the skin effect theory of classical electrodynamics, the internal electric and magnetic fields generated by electromagnetic radiation incident on the metal surface fall to $1/e$ of their surface value in a depth $d = \sqrt{2/\mu\sigma'\omega}$, where d is known as the skin depth. For example, for frequencies characteristic for low temperature thermal radiation, e.g. $\nu \approx 3 \times 10^{12}$ Hz ($\lambda = 100\,\mu$m), and for a high conductivity metal at low temperature such as copper $\sigma' = 10^{10}\,\Omega^{-1}$/cm, the corresponding skin depth is given by $\delta \approx 10^{-6}$ cm or 100 Å. Thus, it is anticipated that the thermal radiation from a low temperature surface should originate from a layer near the surface with a thickness $\approx 10^{-6}$ cm. On the other hand, the correlation length should be on the order of the electron mean free path [10] which for high conductivity metals at low temperature are less than approximately 10^{-3} cm. Thus, the individual regions responsible for generating the thermal radiation field can be considered to be infinitely compared to their lateral extent.

§ Landau and Lifshitz [11, Chap. 4] also provide excellent discussions of the nature of the lateral field and its correlation function, although not to the depth provided by Rytov.

The boundary conditions

For good conductors at low temperature, the boundary conditions for determining the electromagnetic field in the vacuum space adjacent to the surface is

$$\boldsymbol{E} = \tilde{Z}(\boldsymbol{H} \times \boldsymbol{n}) \tag{4}$$

where the surface impedance is given by [12]

$$\tilde{Z}/Z_0 = Z^{*2/3}(1-i\sqrt{3}). \tag{5}$$

The expression for the surface impedance given in equation (4) is for the extreme anomalous skin effect (EASE) region, which is the asymptotic limit for high conductivity and low frequency for the standard anomalous skin effect (ASE) theory, i.e. where Dingles parameter ξ satisfies $|\xi| \gg 1$ [13]. EASE theory is in excellent agreement with ASE theory for good conductors and a range of electromagnetic wave frequencies encountered for blackbody radiation below 50 °K and is mathematically much more tractable than the ASE theory. Assuming Pippard's "standard metal" [14]† with $v_F = 1{\cdot}40 \times 10^6$ m/s and $N = 6{\cdot}0 \times 10^{28}$ e/m^3, the expression for the surface impedance given by equation (5) becomes

$$\tilde{Z}/Z_0 = 3{\cdot}18 \times 10^{-12}\, v^{2/3}(1-i\sqrt{3})$$

or at the frequency maximum given by Wein's displacement law $(\omega/T)_{\max} = 3{\cdot}69 \times 10^{11}$ rad/s °K, the surface impedance is given by

$$\tilde{Z}/Z_0 = 4{\cdot}78 \times 10^{-5}\, T^{2/3}(1-i\sqrt{3}).$$

Thus, on metals having a high conductivity, the surface impedance of the metal surface will satisfy the inequality $|Z/Z_0| \ll 1$.

At the surface of the first conductor in which the lateral field is present, it follows from equation (4) that

$$E_1 + \tilde{Z}H_2 = -K_1 \quad \text{and} \quad E_2 - \tilde{Z}H_1 = -K_2 \tag{6}$$

at $z = 0$, whereas at the second surface where no internal fields are assumed to be present

$$E_1 + \tilde{Z}H_2 = 0 \quad \text{and} \quad E_2 - \tilde{Z}H_1 = 0 \tag{7}$$

at $z = l$. Since the surfaces have a high conductivity and the frequencies are sufficiently low that $|\tilde{Z}| \ll Z_0$, it follows that to a high degree of approximation the boundary conditions

$$E_1 = -K_1 \quad \text{and} \quad E_2 = -K_2 \tag{8}$$

hold at $z = 0$ and the boundary conditions

$$E_1 = 0 \quad \text{and} \quad E_2 = 0 \tag{9}$$

hold at $z = l$. It should be noted immediately that, if the approximate boundary conditions in equation (19) are utilized, the Poynting flux into the second surface must be computed in the same way as for normal modes in a lossy waveguide or cavity resonation, namely [11, p. 291]

$$P(\omega) = \frac{1}{2s}\iint Re(\tilde{Z})H_{\tan}^2\,\mathrm{d}s \tag{10}$$

where H_t is the tangential component of the electromagnetic field at the second surface, i.e. at $z = l$.

The correlation function appearing in equations (2) contains the normal absorption coefficient for the surface and, as the correlation function will appear in our final results, we wish to relate the absorption coefficient to the surface impedance, which as we have previously seen is a function of the properties of the metal. The normal reflectance of a metal is related to its surface impedance by

$$R_N = \left|\frac{Z_0-\tilde{Z}}{Z_0+\tilde{Z}}\right|^2 \tag{11}$$

from which, since the normal absorptance of the surface is given by $\alpha_N = 1-R_N$, we have

$$\alpha_N = \frac{4Z_0\,Re(\tilde{Z})}{|Z_0+\tilde{Z}|^2} \simeq \frac{4\,Re(\tilde{Z})}{Z_0} \tag{12}$$

from which, utilizing equation (5), α_N is defined in terms of the frequency.

Equation of heat transfer

The electromagnetic field vectors in the vacuum space between the metal plates have the general form

$$\boldsymbol{E} = \iint_{-\infty}^{\infty} [\boldsymbol{a}(k_1,k_2)\exp(i\boldsymbol{k}\,.\,\boldsymbol{r}) + \boldsymbol{a}'(k_1,k_2) \times \exp(i\boldsymbol{k}'\,.\,\boldsymbol{r})]\,\mathrm{d}k_1\,\mathrm{d}k_2 \tag{13a}$$

$$\boldsymbol{H} = \iint_{-\infty}^{\infty} [\boldsymbol{k} \times \boldsymbol{a}(k_1,k_2)\exp(i\boldsymbol{k}\,.\,\boldsymbol{r}) + \boldsymbol{k}' \times \boldsymbol{a}'(k_1,k_2)\exp(i\boldsymbol{k}'\,.\,\boldsymbol{r})]\,\mathrm{d}k_1\,\mathrm{d}k_2 \tag{13b}$$

where $\boldsymbol{k}$ represents a wave traveling in the positive z direction where $\boldsymbol{k}'$ represents a wave traveling in the negative z direction. Utilizing the simple boundary conditions given by equation (11) at $z = l$ and the more complex boundary condition given by equation (8) at the source surface we may solve for the amplitude of

†These particular values for the electron density and Fermi velocity closely approximate those for the noble metals. The Fermi velocity for the "standard" metal is within 15 per cent of the value for copper and aluminium. The electron density of the standard metal is equal to that of aluminum but deviates from that of copper by 40 per cent. However, in the expression for $\alpha_N(\omega, T)$, the dependence on N goes at $N^{1/3}$, so we are not sensitive to reasonable percentage deviations in N.

the electromagnetic field vectors, i.e. $a(k_1, k_2)$ and $a'(k_1, k_2)$ in terms of the spatial amplitudes $g(k_1, k_2)$ of the lateral source field. The justification of the use of this more complex boundary condition at $x = 0$ will become apparent later in this analysis. Using these results in equation (13b), we find, using equation (10), that the Poynting vector at the second surface is given by

$$P(k) = \frac{2C\,Re(\tilde{Z})}{Z^2}\iint_{-\infty}^{\infty}\left[\frac{k^2}{|(k+Z_0k_3/\tilde{Z})\exp(ik_3l)+(k-Z_0k_3/\tilde{Z})\exp(-ik_3l)|^2} + \frac{k_3^2}{|(k_3+Z_0k/\tilde{Z})\exp(ik_3l)+(k_3-Z_0k/\tilde{Z})\exp(-ik_3l)|^2}\right]dk_1\,dk_2. \quad (14)$$

This expression gives the unidirectional spectral heat flux between two parallel infinite metal surfaces having a spacing distance l and a surface impedance $\tilde{Z}$. We have further assumed that $\tilde{Z}$ is given by EASE theory which means that the surface impedance is independent of temperature. This allows the net spectral heat flux to be determined by simply applying the principle of superposition.

The total unidirectional heat transfer is given by integrating the expression for the unidirectional spectral heat flux given by equation (16) over all frequencies. Two different contributions are presented—one associated with traveling wave modes and the second associated with the quasi stationary modes of the radiation field. The traveling wave modes correspond to those frequencies defined through the inequality $0 \leqslant k_1^2+k_2^2 \leqslant k^2$. For these modes, k_3 is obviously real and hence these waves propagate freely. The quasi-stationary modes of the thermal radiation field, on the other hand, correspond to those frequencies which are defined through the inequality $k^2 < k_1^2+k_2^2 \leqslant \infty$. For these modes, k_3 is obviously imaginary and therefore these solutions correspond to modes of the radiation field that are exponentially damped in the direction normal to the metal surface.

HEAT-TRANSFER EFFECTS

As has been indicated, the traveling wave portion of the unidirectional heat transfer is defined by values of the propagation vector for the lateral field satisfying the inequality $0 \leqslant k_1^2+k_2^2 \leqslant k^2$. Hence, from equation (16) utilizing equation (5c), the spectral unidirectional heat flux due to traveling waves is given by

$$P_t(k) = \frac{Z_0C\,Re(\tilde{Z})}{16\pi^3\,|\tilde{Z}|^2}\frac{\hbar\omega\alpha_N(\omega)}{\exp(\hbar\omega/KT-1)} \times \iint_{-\infty}^{\infty}\left[\frac{1}{|\cos k_3l+i(Z_0k_3/\tilde{Z}k)\sin k_3l|^2} + \frac{1}{|\cos k_3l+i(Z_0k/Zk_3)\sin k_3l|^2}\right]dk_1\,dk_2. \quad (17)$$

For a given value of k_1 and k_2, as k varies so does k_3, and since at low temperature $|Z_0/\tilde{Z}| \gg 1$ as k varies, the function $P_t(k)$ is sharply peaked at values of $k_3l \approx r\pi$, when r is a positive integer. The function $P_t(k)$ can therefore be considered to be a set of delta-function-like spikes, so that the total unidirectional heat transfer due to traveling waves is simply the sum of the contributions of each of these spikes. If the more complex boundary conditions given by equation (8) had not been utilized in deriving equation (16), then the term $\cos k_3l$ would be absent in equation (17) and the contribution of each spike would be infinite. We determine the contribution of each spike by expanding the terms $\cos k_3l$ in the denominator of equation (17) about $r\pi$ in a narrow region, i.e. $k_3l = r\pi+\zeta l$, thus $\cos k_3l \approx 1$ and $\sin k_3l \approx \zeta l$ where ζ is a new variable. Once a particular spike is considered and the above expansions have been implemented within the integrand, then the limits of integration are extended to infinity; this does not affect the result to any degree, since the main contribution to the spike is in the region where $l \ll 1$, but does lead to a well defined result for the integration. Adopting this approach, the total unidirectional heat flux due to traveling waves can be shown to be given by†

$$\boldsymbol{P}_t = \sum_{\text{spikes}}\int_{\text{spike}} P_t(k)\,dk = \frac{C}{16\pi^2l}\sum_{n_3=0}^{\infty}\iint_{0\leqslant k_1^2+k_2^2\leqslant k^2}\frac{\alpha_N(\omega)\hbar\omega}{\exp(\hbar\omega/KT-1)} \times\left[1+\frac{k_3^2}{k^2}\right]dk_1\,dk_2 \quad (18)$$

where

$$k_3 = \frac{\pi n_3}{l}.$$

For two similar infinite metal surfaces of infinite extent at large spacing distances in the EASE regime, the unidirectional heat transfer is given by

$$\boldsymbol{P}_\infty = \tfrac{1}{2}\varepsilon_H\sigma T^4 \quad (19)$$

where it follows from equations (7) and (14) and the usual integration over the blackbody spectrum that

$$\varepsilon_H = \frac{80}{\pi 4}\Gamma\left(\frac{14}{3}\right)\zeta\left(\frac{14}{3}\right)Z^*\left(\frac{KT}{h}\right)^{2/3}. \quad (20)$$

†To arrive at this form of equation (18) we have utilized the fact that $4[Re(\tilde{Z})]^2/|\tilde{Z}|^2$ is unity which follows from equation (7).

Introducing the nondimensional spacing distance $a = KTl/\hbar c$ and the above result we can write equation (18) in the following nondimensional form

$$P_t/P_\infty = \frac{3}{4a\Gamma(14/3)\zeta(14/3)} \times \sum_{n_3=0}^{\infty} \int_{|n_3/a|}^{\infty} \frac{n^{8/3}}{\exp n - 1}\left(1 + \frac{n_3^2}{a^2 n^2}\right) dn. \quad (21)$$

The nondimensional unidirectional heat transfer P_t/P_∞ due to traveling waves predicted by equation (21) is shown in Fig. 1 as a function of the nondimensional spacing distance a. Departures from classical theory for the traveling wave components occur where nondimensional spacing distance satisfies the inequality $a \leqslant 0{\cdot}1$, i.e. $lT \leqslant 0{\cdot}1$ cm °K. In this nonclassical range, the unidirectional heat transfer due to traveling waves is given by

$$P_t/P_\infty \simeq \frac{3\Gamma(11/3)\zeta(11/3)}{4\Gamma(14/3)\zeta(14/3)a} \quad (22)$$

so that the unidirectional heat transfer due to traveling waves for $a < 0{\cdot}1$ is given by $P_t/P_\infty \simeq 0{\cdot}216/a$. The author has previously derived this result using alternate techniques and discussed the physical reasoning behind this result [15].

For the total heat transfer due to quasi-stationary waves, we have starting from equation (17) that

$$P_q = \int_0^\infty P_q(k)\,dk = \tfrac{1}{8}\int_0^\infty \frac{\hbar\omega}{\exp(\hbar\omega/KT - 1)} \times \left\{ \int_0^\infty \left[\frac{1}{|\cosh \alpha l + (Z_0\alpha/\tilde{Z}k)\sinh \alpha l|^2} + \frac{1}{|\cosh \alpha l + (Z_0 k/Z\alpha)\sinh \alpha l|^2} \right] \alpha\, d\alpha \right\} d\omega \quad (23)$$

where we have utilized the relationship that the factor $Z_0\alpha_N(\omega)Re(Z)/|\tilde{Z}|^2$ appear in equation (17) is unity. The parameter α occurring in equations is defined by $k_3 = i\alpha$ where α is real since in the quasi-stationary region $k_3^2 = k^2 - (k_1^2 + k_2^2) < 0$.

Next in equation (23) we utilize the expression for P_∞ given by equation (19), introduce the changes in variable $x = \hbar\omega/KT$, $\alpha' = \alpha l$, introduce the nondimensional spacing distance $a \equiv KTl/\pi\hbar c$, to obtain the following expression for the unidirectional heat transfer due to quasi-stationary waves

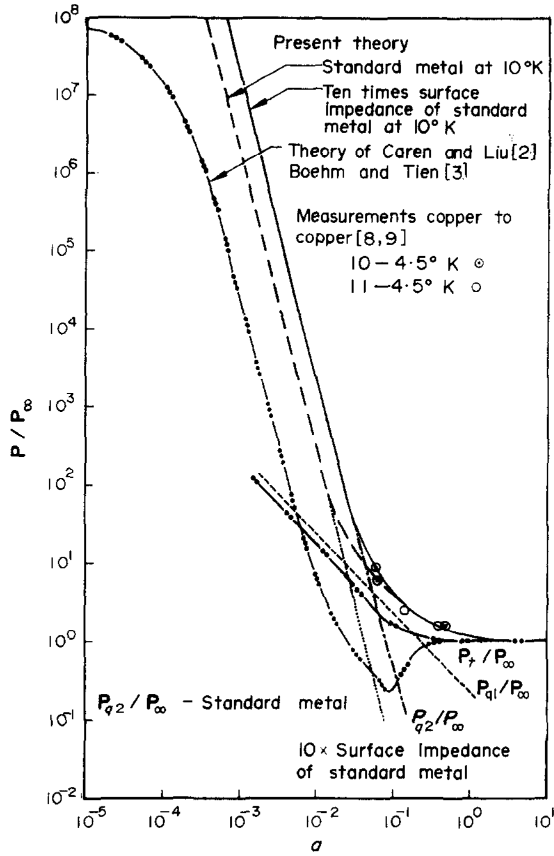

FIG. 1. Nondimensional unidirection radiant heat transfer as a function of nondimensional spacing distance.

We denote the contribution of the first double integral on the right side of equation (24) as P_{q1}/P_∞ and the second double integral as P_{q2}/P_∞. For the "standard metal" the contributions due to these two factors are shown in Fig. 1 for $T = 10$°K. Calculations of these quantities at other values of the parameters Z^* and T show that P_{q1}/P_∞ defines a universal curve independent of Z^* ànd T whereas P_{q2}/P_∞ is dependent on Z^* and T. The superposition of the various heat fluxes P_t/P_∞, P_{q1}/P_∞, and P_{q2}/P_∞ is shown in Fig. 1 for the standard metal at $T = 10$°K. Also shown as isolated data points on this same figure are the limited experimental data points on this same figure are the limited experimental data of Cravahlo, Domoto, and Tien [8] and Domoto, Boehm and Tien [9] taken for copper surfaces near

$$P_q/P_\infty = P_{q1}/P_\infty + P_{q2}/P_\infty = \frac{3}{16\Gamma(14/3)\zeta(14/3)a^2}\int_0^\infty \frac{x}{\exp x - 1} \times \left\{ \int_0^\infty \left[\frac{1}{\left|\cosh \alpha' \frac{(1+i\sqrt{3})h^{2/3}}{4\pi Z^* K^{2/3}} \frac{\alpha'}{aT^{2/3}x^{5/3}} \sinh \alpha'\right|^2} + \frac{1}{\left|\cosh \alpha' + \frac{(1+i\sqrt{3})\pi h^{2/3}}{4Z^*K^{2/3}} \frac{ax^{1/3}}{T^{2/3}\alpha'} \sinh \alpha'\right|^2} \right] \alpha'\, d\alpha' \right\} dx. \quad (24)$$

10°K. Since their data showed an emittance at large spacing distances approximately ten times that given by EASE theory, a second curve has been developed in Fig. 1 $\boldsymbol{P}_{q2}/\boldsymbol{P}_\infty$ for ten times the surface impedance of the "standard metal" to illustrate the effect of more common surface conditions that are met in engineering practice on the heat transfer. We shall show subsequently that when we consider the nondimensional heat flux, only $\boldsymbol{P}_{q2}/\boldsymbol{P}_\infty$ is a function of the surface impedance. Also shown in Fig. 1 are the predictions of the previously discussed theory of Caren and Liu [2] and Boehm and Tien [3]. As previously discussed, this latter theory is based on the assumption of a well characterized isotropic thermal radiation field pervading the metallic medium coupled with radiation tunneling between the medium as developed by the present author using electromagnetic boundary value theory [1]. Rytov [5] has shown, however, that no such radiation field can exist within a highly absorbing medium. It is readily apparent that these earlier theories, although providing results similar in form to the present theory, are quantitatively different and do not show the agreement with experiment provided by the present theory. As can be seen in Fig. 1, the results of the present theory are consistent with the limited experimental data for heat transfer at small spacing distances at 10°K. However, more experimental data are certainly desirable to verify the results of this theory at smaller values of the nondimensional spacing distance. To provide a better understanding of the dependence of the unidirectional heat transfer due to quasi-stationary waves on the temperature, spacing, and surface impedance of the metal surface, we will next provide accurate analytical approximations for the double integrals appearing in equation (24). Consider first the double integral representing the term $\boldsymbol{P}_{q1}/\boldsymbol{P}_\infty$ in equation (24). We are interested in values of the nondimensional spacing parameter $a \lesssim 10^{-1}$. Therefore, in the integral with respect to α', the term $h^{2/3}/4\pi Z^*K^{2/3}a \gg 1$; in fact for the "standard metal" where $a \simeq 10^{-1}$, we have $h^{2/3}/4\pi Z^*K^{2/3}a \simeq 10^5$. Thus, as is obvious in a self-consistent type of argument, the integral with respect to α' has its significant contributions for $\alpha' \approx 1$, we have that the principal contributions to the integral with respect to α' are for α' small. Thus, we approximate $\sinh\alpha'$ in the denominator by α' and the term $\cosh\alpha'$ by unity. A straightforward integration of the resulting expression yields

$$\boldsymbol{P}_{q1}/\boldsymbol{P}_\infty \simeq \frac{\pi^2\Gamma(11/3)\zeta(11/3)}{4\sqrt{3}\Gamma(14/3)\zeta(14/3)a} \quad \text{for } a \lesssim 1. \quad (25)$$

Numerical evaluation of the corresponding double integral in equation (24) shows that this approximation is good to 3 per cent in the range for which $a \lesssim 10^{-1}$. It indicates further that this contribution to the heat transfer due to quasi-stationary waves expressed in terms of the above nondimension parameters is independent of the variables of surface impedance and temperature. Its variation with spacing distance exactly reflects the same inverse spacing distance dependence as the contribution due to traveling waves for $a \leqslant 10^{-1}$. Consider next the expression for $\boldsymbol{P}_{q2}/\boldsymbol{P}_\infty$ given by equation (24), since in the integral with respect to α' the parameter $\pi h^{2/3}/4Z^*K^{2/3} \gg 1$; in fact for the "standard metal" $\pi h^{2/3}/4Z^*K^{2/3} \simeq 3{\cdot}3 \times 10^4$, for values of a such that $\pi h^{2/3}a/4Z^*K^{2/3} \gg 1$ we neglect the term $\cosh\alpha'$ in the integral with respect to α'. Thus, for the "standard metal" where $a \gtrsim 10^{-3}$ or in general when $a \gtrsim 40Z^*K^{2/3}/\pi h^{2/3}$, we find that

$$\boldsymbol{P}_{q2}/\boldsymbol{P}_\infty = \frac{9Z^*\Gamma(4/3)\zeta(4/3)\zeta(3)}{8\pi^2\Gamma(14/3)\zeta(14/3)a^4}\left(\frac{KT}{h}\right)^{2/3}. \quad (26)$$

Numerical evaluation of the double integral for $\boldsymbol{P}_{q2}/\boldsymbol{P}_\infty$ appearing in equation (24) shows that equation (26) is accurate to better than 10 per cent over the ranges of a indicated above. Where a is quite small, i.e. $\pi h^{2/3}a/4Z^*K^{2/3} \ll 1$, we may neglect the corresponding terms in the denominator of the integral with respect to α', thereby retaining only the term $\cosh\alpha'$ in the denominator. Thus, for the "standard metal" for $a \lesssim 10^{-5}$ or in general for we find that

$$\boldsymbol{P}_{q2}/\boldsymbol{P}_\infty \simeq \frac{3\Gamma(2)\zeta(2)\ln 2}{16\Gamma(14/3)\zeta(14/3)Z^*a^2}\left(\frac{h}{KT}\right)^{2/3}. \quad (27)$$

Numerical evaluation of the double integral for $\boldsymbol{P}_{q2}/\boldsymbol{P}_\infty$ appearing in equation (24) shows that equation (27) is accurate to better than 10 per cent over the ranges of a indicated above. As can be seen from Fig. 1, the component $\boldsymbol{P}_{q2}/\boldsymbol{P}_\infty$ of the quasi-stationary radiation field dominates the heat transfer for $a \lesssim 10^{-2}$. Equations (26) and (27) show that in distinction to the components $\boldsymbol{P}_t/\boldsymbol{P}_\infty$ and $\boldsymbol{P}_{q1}/\boldsymbol{P}_\infty$, the components $\boldsymbol{P}_{q2}/\boldsymbol{P}_\infty$ of the heat transfer is dependent on the surface impedance and surface temperature. Its dominance at small values of a comes from the strong inverse power dependence of the magnitude of this component of the heat transfer with spacing.

SUMMARY AND DISCUSSION

This analysis has addressed radiation heat-transfer effects between closely spaced metal surfaces at low temperature considering as the source term the small-scale centers of thermally induced polarization at the metal surface. According to Leontovich and Rytov, these centers have a spatial extent on the order of the electron mean free path in the metal. For a metal at low temperature, the mean free path Λ is long; and since the skin effect is well developed, the skin depth d is quite small, so that $d/\Lambda \ll 1$. Thus, to an external observer, the source regions have finite lateral extent but no depth. Hence, the polarization giving rise to the external field is in the plane of the surface and in turn gives rise to a lateral electron field at the surface of the metal. The standard electromagnetic boundary conditions then demand the presence of an equal lateral field exterior to the metal. Rytov has derived the correlation function for the lateral field and shown that the propagation vector associated with the lateral field extends over the range $0 \leqslant k_1^2+k_2^2 < \infty$. For each wavelength of the thermal radiation field in the vacuum, therefore, this lateral source field gives rise in the vacuum region bounding the metal to both real traveling waves for $0 \leqslant k_1^2+k_2^2 \leqslant k^2$, i.e. k_3 is real, and to quasi-stationary waves for $k^2 < k_1^2+k_2^2 \leqslant \infty$, i.e. k_3 is imaginary. The quasi-stationary field is a near surface field since k_3 is imaginary, and therefore this component is experimentally damped with increasing distance from the surface. The traveling wave field, on the other hand, gives rise to the thermal radiation observed at large distances from the metal surface, i.e. to the classically observed thermal radiation phenomena. The source strength of the thermally induced lateral field is determined by equating the formally derived Poynting flux from the surface derived in terms of the undetermined strength of the source field to the classical radiant emission $P(\omega) = \pi\varepsilon_{H\omega}I_{0\omega}$ from the surface. In this latter expression, $\varepsilon_{H\omega}$ is determined by the electrodynamic characteristics of the metal as given by EASE theory. The present results are limited, for convenience, to the analysis of the unidirectional heat flux from a metal surface at low temperature to a second low temperature metal surface in close proximity. However, since in the EASE region the surface impedance is independent of temperature, net radiant heat transfer can be determined by a straightforward application of the superposition principle. Starting with the temperature-dependent correlation functions for the lateral field and using the standard electromagnetic boundary conditions, the amplitudes of the electric and magnetic field vectors are determined. In this analysis, only one wall is assumed to be a source. The resulting spectral heat flux to the opposing (receiving) wall is determined by the application of the Poynting theorem at its surface. The integral of the resulting expression over all frequencies provide the total unidirectional heat flux. The expression for the total unidirectional heat flux contains three terms, a contribution due to traveling waves [see equation (21)] and two distinct contributions due to quasi-stationary waves [see equation (24)]. The contributions due to quasi-stationary waves provide negligible contributions to the unidirectional heat flux at large spacing distances while the contribution due to traveling waves provides the classical heat transfer equation, i.e. $\boldsymbol{P}_t/\boldsymbol{P}_\infty = 1$, for nondimensional spacing distances satisfying the inequality $a \gtrsim 1$, i.e. $l \gtrsim 2{\cdot}5\lambda_m$. In the range of nondimensional spacing distances bounded approximately by $10^{-1} \lesssim a \lesssim 1$, the unidirectional heat flux increases over the value at infinite spacing distances with approximately equal contributions (see Fig. 1) due to traveling waves and the component of the heat transfer due to quasi-stationary waves defined as P_{q1}/P_∞ [see equation (24)]. These contributions, when written in the nondimensional form graphically illustrated in Fig. 1, are independent of surface temperature and surface impedance. For nondimensional spacing distances satisfying the inequality of $\lesssim 10^{-1}$, both of these contributions increase indirect inverse proportion to the nondimensional spacing distance [see equations (22) and (25)]. For smaller nondimensional spacing distances, i.e. $a \lesssim 10^{-2}$, the unidirectional heat flux is dominated by a shorter range component of the contribution, due to quasi-stationary waves, which is defined by the term $\boldsymbol{P}_{q2}/\boldsymbol{P}_\infty$ appearing in equation (24). When written in the nondimensional form as depicted in Fig. 1, this term is dependent on the surface temperature and surface impedance [see equations (26) and (27)]. To provide a physical insight into the above summary of the major findings of this paper, first consider the contribution of the traveling wave component to the unidirectional heat flux. The traveling waves emitted by a free surface undergo multiple reflections when a second parallel metal surface is present. Providing the conductivity is high, this will lead to a buildup in the field strength in the cavity for those wavelengths and directions of propagation that lead to antinodes at the metal surfaces, i.e. to normal modes in the cavity. Consider now the energy density in a rectangular cavity two large dimensions, l_1 and l_2, and one small dimension l. Assume further that the temperature of the cavity is such that $\lambda_m \gg l$. The normal modes of the electromagnetic field in the cavity are given by the relationship [16]

$$\begin{aligned}\omega^2_{n_1,n_2,n_3} &= (2\pi)^2/\lambda^2_{n_1,n_2,n_3} \\ &= \pi^2c^2(n_1^2/l_1^2+n_2^2/l_2^2+n_3^2/l_3^2)\end{aligned} \tag{28}$$

where the integers n_1, n_2, n_3 are restricted such that two or more of the integers n_1, n_2, n_3 must be nonzero. Therefore, it follows that all modes for which $n_3 \neq 0$ essentially do not contribute to the thermal radiation density since $\lambda_{n_1,n_2,n_3} \ll \lambda_m$. Thus, the modes that do contribute to the thermal radiation density are those for which $n_3 = 0$. There is a large number of such modes, since for large values of l_1 and l_2 there will be a large number of combinations of integers n_1 and n_2 for which $\lambda_{n_1,n_2,0} \approx \lambda_m$. Under these conditions, as the small dimension l of the box tends toward zero, the number and, therefore, the total energy associated with these modes remains fixed, yet the volume containing this total radiant energy decreases as l and thus the corresponding energy density increases as $1/l$. Since the breakpoint for the energy density should occur where $l \approx \lambda_m$, we should expect for small values of l that $U \approx U_\infty(\lambda m/l)$ or $U \approx U_\infty(0{\cdot}4/a)$. Now the unidirectional heat flux to the second wall is given approximately by $P = 1/4\alpha c U$ where α is the "spectral average" of the absorbtivity of the walls or $P/P_\infty \simeq U/U_\infty$, so that we would anticipate that $P/P_\infty \approx 0{\cdot}4/a$.

Our rigorous calculation provided by equation (22) gives $P/P_\infty = 0{\cdot}216/a$. Thus, the above discussion provides us with a good insight into the behavior of the traveling wave contribution to the unidirectional heat flux at small spacing distances where correlation in the source term has not been considered. Consider now the contribution to the unidirectional heat flux due to quasi-stationary waves. Consider the magnetic field vector near an isolated metal surface defined in terms of the strength of the later source field using the simple boundary condition given by equation (8); we have then

$$H = \frac{1}{Z_0 k}\iint_{-\infty}^{\infty} [k \times a(k_1, k_2)] \exp(ik \, . \, r) \, dk_1 \, dk_2 \quad (29)$$

where

$$a_1 = -g_1, \quad a_2 = -g_2, \quad a_3 = \frac{k_1 g_1 + k_2 g_2}{k_3}.$$

We choose to compute the quantity $(H \, . \, H^*)$ since this is both a measure of the field strength and would be proportional to the heat flux [see equation (10)] transferred to a second parallel metal surface in the vicinity of the source surface where multireflection effects are not of significance. For a lateral source as provided by equation (2a), we find that for the contribution due to quasi-stationary waves, i.e. $k_1^2 + k_2^2 > k^2$

$$(H \, . \, H^*)_{\tan} = \frac{2\pi C}{Z_0^2 k^2}\int_0^\infty \left(\frac{k^4}{\alpha^2} + \alpha^2\right) \exp(-2\alpha l) \alpha \, d\alpha \quad (30)$$

where we have used the notation $k_3 = i\alpha$. Consider now the first item in the expression for $(H . H^*)_{\tan}$, which corresponds in the present approximations to the term giving rise to the component P_{q1}/P_∞ in equation (24), we have that

$$(H \, . \, H^*)_{\tan_1} = \frac{2\pi k^2 C}{Z_0^2}\int_0^\infty \frac{\exp(-\alpha l)}{\alpha} \, d\alpha. \quad (31)$$

This integral is divergent because we have utilized the approximate boundary condition given by equation (8) in order to derive simple expressions which lead more readily to physical interpretations rather than the exact boundary given by equation (6) in deriving this result. The more exact boundary condition essentially leads to a quite small but finite lower bound on α [from equation (24)]; for example, we can find that effectively the lower bound on α is given by $\alpha_L \approx 10^{-2}$) which we will assume to be present in equation (31). Hence, the principal contributions to the integral will be for $\alpha > \alpha_L$ but at the same time for $\alpha \simeq \alpha_L$, under this condition $\exp(-\alpha l) \simeq 1$. This latter approximation follows since, as can be deduced from Fig. 1, for $\alpha_L \approx 10^{-2}$ the range of values of l for which P_{q1}/P_∞ make significant contributions will correspond to $\alpha_L l < 1$. Thus, in the range of values of $l \lesssim 1/\alpha_L \approx$ several centimeters we have that

$$(H \, . \, H^*)_{\tan_1} \simeq \frac{2\pi k^2 C}{Z_0^2}\int_{\alpha_L}^\infty \frac{d\alpha}{\alpha}. \quad (32)$$

The field strength is thus independent of distance from the metal surface providing $l \lesssim 1/\alpha_L$. Also note that a small but approximately fixed range of values of the lateral propagation vector in the lateral field contribute, namely, those values for which $(k_1^2 + k_2^2) > k^2$ but for which also $(k_1^2 + k_2^2) \simeq k^2$, i.e. those values for which $k^2 - (k_1^2 + k_2^2) \simeq \alpha_L^2$. As a second metal surface is brought near the source surface due to the strong unattenuated nature of this term for $l \lesssim 1/\alpha_L$, the field strength will be built up in direct proportion to the numbers of multiple reflections or equivalently proportional to $1/l$. Analogous to the traveling wave case, the energy density due to this component of the quasi-stationary field will increase in direct proportion to $1/l$, i.e. indirect inverse proportion to the nondimensional spacing distance, and will build up to a value independent of the strength of the source term. Hence the unidirectional heat-transfer effects due to this term in the quasi-stationary field should be directly proportional to the inverse of the nondimensional spacing distance and depend only on the absorptance (emittance) of the receiving surface. Thus, written in nondimensional terms, this term should lead to a unidirectional heat flux which is independent of surface impedance. This is confirmed by the exact evaluation of the term P_{q1}/P_∞ in equation (25) which are depicted in Fig. 1. Consider now the term P_{q2}/P_∞ in equation (24). The analogous term in our approximate expression for the magnetic field strength due to an isolated surface given in equation (30) is

$$(H \, . \, H^*)_{\tan_2} = \frac{2\pi C}{Z_0^2 k^2}\int_0^\infty \exp(-2\alpha l)\alpha^3 \, d\alpha. \quad (33)$$

Equation (33) can be integrated to yield

$$(H \, . \, H^*)_{\tan_2} = \frac{3\pi C}{4 Z_0^2 k^2}\frac{1}{l^4}. \quad (34)$$

The integral in equation (33) can be interpreted physically as a superposition of a continuum of quasi-stationary waves with a high density at large values of α (increasing as α^3) which contribute significantly only if $\alpha l \lesssim$ of 1. Thus, with decreasing spacing distance the number of waves able to reach over to the second surface is the dominant factor and not multiple reflections of the less spectrally dense, more weakly attenuated waves. Thus, a volumetric effect in the energy density is not involved in relating equation (30) to the unidirectional heat transfer. The magnitude of the unidirectional heat-transfer effects are dependent in this case on the source strength (which is directly proportional to the surface impedance) and the absorbtivity of the second surface (which is also directly dependent on the surface impedance). Thus, written in our standard unidirectional form, we would expect $\boldsymbol{P}_{q2}/\boldsymbol{P}_{\infty}$ to be dependent on surface impedance.

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RAYONNEMENT THERMIQUE ENTRE SURFACES METALLIQUES TRES PROCHES, A BASSE TEMPERATURE, DU AUX COMPOSANTES PROGRESSIVES ET QUASI-STATIONNAIRES DU CHAMP DE RADIATION

Résumé—On analyse le rayonnement à basse température entre des surfaces métalliques parallèles très proches. La théorie considère les champs électriques fluctuants thermiquement induits à la surface du métal, semblables à ceux qui sont responsables du bruit de Nyquist, et sources du champ de rayonnement thermique dans l'espace vide entre les métaux. On montre qu'il existe dans l'espace vide à la fois une onde progressive et une onde quasi stationnaire. Les vecteurs associés aux champs électrique et magnétique qui existent dans l'espace vide sont déterminés en utilisant la théorie classique des frontières électromagnétiques et les flux thermiques unidirectionnels sont calculés en utilisant le théorème de Poynting. Les flux thermiques correspondent à des effets importants et insoupçonnés quand le produit de la distance l à la température des surfaces T est inférieur à 1 cm K. Aux faibles valeurs de l le flux thermique varie tout d'abord comme l'inverse de l, puis quand $lT \leqslant 10^{-2}$ cm K, le transfert unidirectionnel croît proportionnellement à la quatrième puissance de ce produit. On constate que ces résultats sont conformes aux mesures expérimentales antérieures.

WÄRMESTRAHLUNG ZWISCHEN DICHT GEGENÜBERLIEGENDEN METALLISCHEN OBERFLÄCHEN BEI TIEFEN TEMPERATUREN AUFGRUND DER WANDERNDEN UND QUASI-STATIONÄREN KOMPONENTEN DES STRAHLUNGSFELDS

Zusammenfassung—Es wird die Wärmeübertragung durch Strahlung bei tiefen Temperaturen zwischen dicht gegenüberliegenden, parallelen metallischen Oberflächen untersucht. Die Theorie berücksichtigt thermisch induzierte, wechselnde elektrische Felder an den metallischen Oberflächen wie jene, die das Nyquist-Rauschen verursachen, als Quelle für das Wärmestrahlungsfeld in dem leeren Raum zwischen den Metallplatten. Es wird gezeigt, daß sowohl wandernde als auch quasi-stationäre Wellenkomponenten des Wärmestrahlungsfelds als Folge dieser Quellen im Vakuumbereich existieren. Die diesen Feldern im leeren Raum zugeordneten elektrischen und magnetischen Vektorfelder sind unter Anwendung der geläufigen elektromagnetischen Randtheorien abgeleitet, und die einseitig gerichteten Wärmeflüsse sind unter Verwendung des Poynting-Theorems berechnet. Es wird gezeigt, daß die resultierenden Wärmeströme zu sehr ungewöhnlichen Auswirkungen der Wärmeübertragung in Beziehung stehen, wenn das Produkt der Entfernung l und der Temperatur T kleiner als 1 cm K ist. Bei kleinen Entfernungen nimmt der Wärmefluß umgekehrt proportional zur Entfernung zu, und wenn $l \cdot T \lesssim 10^{-2}$ cm K ist, nimmt der ungerichtete Wärmefluß umgekehrt proportional zur 4. Potenz dieses Produkts zu. Es wird berichtet, daß die Ergebnisse dieser Theorie mit vorangegangenen experimentellen Messungen übereinstimmen.

ЛУЧИСТЫЙ ТЕПЛОПЕРЕНОС ПРИ НИЗКОЙ ТЕМПЕРАТУРЕ МЕЖДУ БЛИЗКО РАСПОЛОЖЕННЫМИ ДРУГ К ДРУГУ МЕТАЛЛИЧЕСКИМИ ПОВЕРХНОСТЯМИ ЗА СЧЁТ ПОДВИЖНЫХ И КВАЗИСТАЦИОНАРНЫХ СОСТАВЛЯЮЩИХ ПОЛЯ ИЗЛУЧЕНИЯ

Аннотация — Анализируются эффекты лучистого теплопереноса между близко расположенными друг к другу параллельными металлическими поверхностями при низкой температуре. Термически индуцированные флуктуационные электрические поля на металлической поверхности, аналогичные полям, создающим шумы Найквиста, рассматриваются как источники поля теплового излучения в вакуумном пространстве между металлическими поверхностями. Показано, что благодаря этим источникам в вакуумной зоне имеют место как бегущая волна поля теплового излучения, так и квазистационарная волна. С помощью общепринятой электромагнитной теории пограничного слоя выводятся векторы электрического и магнитного полей, характерные для полей в вакуумном пространстве, а посредством теоремы Пойнтинга рассчитываются результирующие однонаправленные тепловые потоки. Показано, что результирующие тепловые потоки соответствуют в высшей степени неклассическим эффектам теплопереноса, когда произведение расстояния между поверхностями l на температуру поверхности T оказывается меньше 1 см°К. При небольших зазорах тепловой поток сначала расчёт обратно пропорционально величине зазора, а затем при $lT \lesssim 10^{-2}$ см°К однонаправленный теплоперенос возрастает обратно пропорционально четвертой степени этого произведения. Показано, что эта теория соответствует экспериментальным данным ранее проведенных измерений.